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Transient conjugated heat transfer to laminar flow in a tube or channel

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INTRODUCTION

IN ACTUAL practice there are two types of conjugated heat transfer problems involving mutual coupling of wall and fluid temperatures. If there is heat conduction in a wall, coupling must often be taken into account since the heat transfer coefficient is not known *a priori* but is dependent on the streamwise variation of wall temperature. Other conjugated problems which require different kinds of solution schemes are those where transients can occur. The need for conjugated treatment is great especially in the case of a laminar flow where the thermal history effect on the heat transfer coefficient is appreciable. Turbulent flows can, on the other hand often be solved in actual practice by neglecting the coupling of fluid and wall temperature fields since the effect of streamwise temperature variations on the heat transfer coefficient is not very strong.

One of the first papers concerning the effect of conduction in a wall on a laminar heat transfer is the work of Mori *et al.* [1], which has also been extended to include turbulent flows by Sakakibara and Endoh [2]. The Japanese researchers just mentioned have also published other papers on conduction–convection heat transfer. The most recent works published concerning the effect of wall conduction are by Barozzi and Pagliarini [3] and Wijeysundera [4].

The next stage of complexity is reached when transient heat transfer is considered. Solutions for transient problems have been obtained only by making approximations, e.g. slug flow assumption. The first work to use an actual velocity profile in a laminar flow is that of Sucec [5]. He approximates it with a linear velocity profile which is correct at the thermal entrance region. In that paper also a reference list of earlier works where different kinds of approximations are used, is given. Later on Sucec and Sawant extended the quasi-steady method of ref. [5] to include the actual quadratic velocity profile of a parallel channel and applied this method, using a Laplace-transform technique, to an insulated channel with a sinusoidal inlet temperature variation [6]. The most recent work on transient laminar heat transfer of a channel is the numerical solution of Sucec [7].

In this paper a method for solving transient problems of a laminar channel or tube flow is presented and the results are given for a step change of inlet fluid temperature for the cases where wall and fluid temperatures are constant initially. This paper is an extension of the method of Sucec for a channel flow and results are compared with those of Sucec. Calculated results are given in the case of a tube flow and these are compared with measured results.

GOVERNING THEORY AND SOLUTION PROCEDURE

The problem considered in this note is shown in Fig. 1, which shows a tube or a channel. The convective heat transfer coefficient to the surroundings is assumed to be known. Initially the fluid and wall are at the same temperature T_∞ as the surroundings. At some instant the fluid temperature undergoes a step change to a new value T_0 while still remaining a fully developed velocity profile. The problem is to find wall and bulk fluid temperatures that depend on both the streamwise coordinate x and time t.

First consider a steady-state heat transfer. If in Fig. 1 the inside surface temperature of a wall is known, the corresponding heat flux in a round tube with a laminar flow can be obtained from the well-known equation

$$q(x^*) = \frac{4k}{d}\int_0^{x^*} \sum_{n=0}^{\infty} G_n \, e^{-\lambda_n^2 (x^*-\xi)} \, dT_w(\xi) \tag{1}$$

where the eigenfunctions G_n and eigenvalues λ_n can be found in the literature [8, 9]. For a channel flow the corresponding analogous equation to equation (1) is also found in the literature. Now the question is, can equation (1) be used also in transient heat transfer.

Sucec, when developing his quasi-static method for a laminar channel flow, approached the problem from a mathematical viewpoint [5]. He derived equations that are valid for a slug flow and approximated a real flow by using the connection between heat flux and surface temperature that is based on a linear velocity profile and is correct at the thermal entrance region. Actually this means that he assumed convective heat transfer to be steady at a known instant. As a matter of fact as Sucec has pointed out, this is the same kind of treatment that has been used in refs. [10, 11] for transient external heat transfer. Physically this kind of treatment means that the heat transfer process is slow, so that the heat capacity effect of a fluid can be neglected when compared with the convective heat flux at the wall, but the thermal history effect is taken into account.

When the above approach is used, the wall temperature of a tube in Fig. 1 is governed by

$$\rho_w c_{pw} dl \frac{\partial T_w}{\partial t} = -4k \int_0^{x^*} \sum_{n=0}^{\infty} G_n \, e^{-\lambda_n^2 (x^*-\xi)} \, dT_w(\xi) - d_u h(T_w - T_\infty). \tag{2}$$

In equation (2) it is assumed that the wall temperature T_w is lumped in the cross-stream direction and for a thick-walled tube l is an equivalent wall thickness so that wall heat capacity is correctly evaluated. For a channel flow the corresponding equation for equation (2) is also easily derived and it is given in ref. [6] for an insulated channel. A limit where equation (2) is valid can be only obtained by tedious numerical calculations, i.e. by solving the energy differential equation numerically, or by making experimental measurements.

In the solving procedure it is assumed that wall tem-

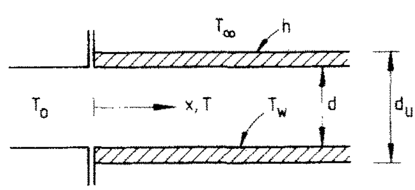

FIG. 1. Schematic presentation of a tube or channel.

NOMENCLATURE

b	half thickness of channel
c_p, c_{pw}	fluid and wall specific heat, respectively
d, d_u	inside and outside tube diameter, respectively
G_n	eigenfunction
h	heat transfer coefficient to the surroundings
k	thermal conductivity of fluid
l	wall thickness of tube or channel
q	wall heat flux
t	time
u_m	average fluid velocity
T_0	fluid temperature at inlet after step change
T_m	local bulk temperature of fluid
T_w	local wall temperature
T_∞	initial wall and fluid temperature
x	streamwise coordinate from inlet
x^*	non-dimensional distance of tube, $kx/\rho c_p d^2 u_m$
x_c^*	non-dimensional distance of channel, $kx/\rho c_p b^2 u_m$
X^*	non-dimensional variable of tube, $k\tau/\rho_w c_{pw} dl x^{*1/3}$
X_c^*	non-dimensional variable of channel, $k\tau/\rho_w c_{pw} bl x_c^{*1/3}$.

Greek symbols

θ_m	non-dimensional fluid temperature, $(T_m - T_\infty)/(T_0 - T_\infty)$
θ_w	non-dimensional wall temperature, $(T_w - T_\infty)/(T_0 - T_\infty)$
λ_n	eigenvalue
ξ	dummy non-dimensional distance
ρ, ρ_w	density of fluid and wall, respectively
τ	modified time, $t - x/u_m$.

perature remains constant during a small time increment Δt, so that the change in wall temperature during Δt can be obtained from equation (2). The integral of equation (2) is easily calculated if wall temperature is approximated by a series of straight lines. If the length of each reach is Δx^*, the integration to the point $x^* = i\Delta x^*$ gives

$$\int_0^{x^*} \sum_{n=0}^{\infty} G_n \, e^{-\lambda_n^2 (x^* - \xi)} \, dT_w(\xi) = \Delta T_0 \sum_{n=0}^{\infty} G_n \, e^{-i\lambda_n^2 \Delta x^*} + \sum_{j=1}^{i} k_{(i+1-j)} \sum_{n=0}^{\infty} \frac{G_n}{\lambda_n^2} e^{-j\lambda_n^2 \Delta x^*} (e_n^{\lambda \Delta x^*} - 1) \quad (3)$$

where $k_j = (T_{w,j} - T_{w,j-1})/\Delta x^*$ is the streamwise temperature gradient and ΔT_0 is the temperature difference between fluid and wall at the tube inlet.

When a new wall temperature distribution is calculated, it is substituted into the right-hand side of equation (2), and so on. When the wall temperature is known at a given instant t, the corresponding heat flux to the fluid is obtained from equation (1) and the bulk mean temperature of a fluid is calculated from

$$T_m(x^*, t) = T_0 + \frac{4}{\rho c_p u_m d} \int_0^{x^*} q(\xi, t) \, d\xi \quad (4)$$

where time t is included in order to emphasize that one is dealing with a transient problem although convection is calculated from steady-state equations.

RESULTS

First the results of an insulated channel are given. Figure 2 shows the non-dimensional wall temperature of a step change in fluid inlet temperature when the fluid and wall are initially at the same constant temperature. Within drawing accuracy the result is the same as the quasi-steady method of Sucec except that the non-dimensional variable X_c^* must be multiplied with a constant 0.72856 in order to get the same variable as Sucec used. This is slightly surprising because Sucec used a linear velocity profile that is valid at the channel inlet, when on the other hand the connection between heat flux and wall temperature used in this note, i.e. equation (1), is valid everywhere. In non-dimensional variable X_c^* the time delay of flow from the channel inlet has been taken into account approximately by using time variable $\tau = t - x/u_m$ instead of t.

If the wall heat capacity is large compared with the heat capacity of the fluid, so that the heat transfer process is slow, the result of Fig. 2 is valid. The validity limit of the universal result in Fig. 2 can be obtained only by numerical calculations or measurements. According to Sucec the deviation of quasi-static results from the results obtained using a finite-difference method is small if the parameter $a = \rho c_p b/\rho_w c_{pw} l$ is smaller than 1.

From a practical point of view and for applications it would be very useful if the effect of outside convection heat transfer could be included in a non-dimensional presentation of Fig. 2. The referees of this note kindly suggested that perhaps a parameter $hbx_c^{*1/3}/k$ would give a family of response functions that show the dependence of the outside heat transfer coefficient on wall temperature.

The wall temperature of an insulated tube is shown in Fig. 3. It can be seen that also in this case the general non-dimensional presentation of a wall temperature can be found

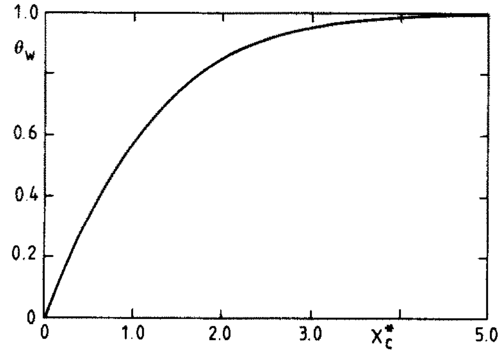

FIG. 2. Response of a channel wall temperature for a step change in inlet temperature.

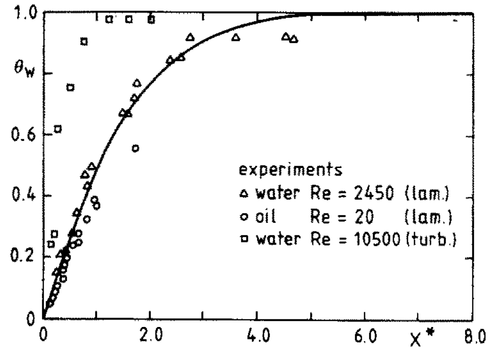

FIG. 3. Calculated response of a tube wall temperature for a step change in inlet temperature and comparison with measured data.

for a step change in fluid inlet temperature. The non-dimensional variable X^* now includes, as in the case of a channel, time and space coordinates, geometry and thermal properties of fluid and wall. Figure 3 also includes wall temperatures measured by the author. The test installation consisted of an insulated steel pipe with an internal diameter equal to 10 mm and a wall thickness of 3.0 mm. The total length of the tube was about 6.5 m and experiments were made using water and oil as the fluid.

Wall temperatures were obtained using thermocouples and flow velocities were determined by measuring the mass of a fluid during a given time interval. The pipe was connected to the reservoir of a heated fluid and a transient was realized when the heated fluid flowed through the insulated pipe, which was at the ambient temperature. In this test installation velocity and temperature profiles developed simultaneously, but due to the large Prandtl numbers of fluids experimental results are comparable with theoretical ones, which are based on a fully developed velocity profile.

It can be seen from Fig. 3 that the theoretical curve is in reasonable agreement with measured data when the velocity profile is laminar. When looking at the results for oil a systematic discrepancy can be seen. The reason may be that in a non-dimensional variable x^* a modified time $\tau = t - x/u_m$ has been used and this may cause the deviation when velocity is very small.

In measurements the ratio of thermal energy capacity of the fluid to that of the wall $a = \rho c_p d^2/\rho_w c_{pw}(d_u^2 - d^2)$ was 0.765 for water and 0.275 for oil, respectively.

In Fig. 3 there are also some measured results when the velocity profile is turbulent. These results are also in good agreement with an analytical solution that is obtained by using the Laplace-transform technique and assuming a heat transfer coefficient of a fully developed turbulent flow [12]. This analytical solution is not shown in Fig. 3, because it cannot be presented by using the non-dimensional variable X^*.

CONCLUSIONS

Transient heat transfer in a tube or channel of finite wall heat capacity must often be solved, especially in the case of a laminar flow, as a conjugated problem where the thermal fields of a wall and fluid are coupled together. In this paper a quasi-static method is presented for transient heat transfer when in a tube or channel the velocity profile is fully developed. The method is based on the assumption of a slow heat transfer process, so that during a small-time increment only the effect of varying wall temperature needs to be taken into account and the fluid heat capacity effect can be ignored. In actual practice this means that wall heat capacity is large compared with that of a fluid. The validity limits of the method must be found experimentally or by using for instance a finite-difference method.

The important feature of the method presented is its simplicity. It is almost an analytical method of which the results can be presented universally for an insulated tube or channel. The method was verified by comparing the results with those presented in the literature and obtained using other methods. In the case of a tube calculated results were compared with measured data.

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